1. Each weekday Alan drives to work. On his journey, he goes over a level crossing. Sometimes he has to wait at the level crossing for a train to pass.

- W is the event that Alan has to wait at the level crossing.
- L is the event that Alan is late for work.

You are given that $P(L|W) = 0.4$, $P(W) = 0.07$ and $P(L \cup W) = 0.08$.

i. Calculate $P(L \cap W)$.

[2]

ii. Draw a Venn diagram, showing the events L and W. Fill in the probability corresponding to each of the four regions of your diagram.

[3]

iii. Determine whether the events L and W are independent, explaining your method clearly.

[3]

2. Jenny has six darts. She throws darts, one at a time, aiming each at the bull's-eye. The probability that she hits the bull's-eye with her first dart is 0.1. For any subsequent throw, the probability of hitting the bull's-eye is 0.2 if the previous dart hit the bull's-eye and 0.05 otherwise.

i. Illustrate the possible outcomes for her first, second and third darts on a probability tree diagram.

[4]

ii. Find the probability that

A. she hits the bull's-eye with at least one of her first three darts,

[3]

B. she hits the bull's-eye with exactly one of her first three darts.

[4]

iii. Given that she hits the bull's-eye with at least one of her first three darts, find the probability that she hits the bull's-eye with exactly one of them.

[3]

Jenny decides that, if she hits the bull's-eye with any of her first three darts, she will stop after throwing three darts. Otherwise she will throw all six darts.

iv. Find the probability that she hits the bull's-eye three times in total.

[4]

3. Each weekday, Marta travels to school by bus. Sometimes she arrives late.

- L is the event that Marta arrives late.
- R is the event that it is raining.

You are given that $P(L) = 0.15$, $P(R) = 0.22$ and $P(L \mid R) = 0.45$.

i. Use this information to show that the events L and R are not independent.

[1]

ii. Find $P(L \cap R)$.

[2]

iii. Draw a Venn diagram showing the events L and R, and fill in the probability corresponding to each of the four regions of your diagram.

[3]

4. Candidates applying for jobs in a large company take an aptitude test, as a result of which they are either accepted, rejected or retested, with probabilities 0.2, 0.5 and 0.3 respectively. When a candidate is retested for the first time, the three possible outcomes and their probabilities remain the same as for the original test. When a candidate is retested for the second time there are just two possible outcomes, accepted or rejected, with probabilities 0.4 and 0.6 respectively.

i. Draw a probability tree diagram to illustrate the outcomes.

[3]

ii. Find the probability that a randomly selected candidate is accepted.

[2]

iii. Find the probability that a randomly selected candidate is retested at least once, given that this candidate is accepted.

[3]

5. Measurements of sunshine and rainfall are made each day at a particular weather station. For a randomly chosen day,

- R is the event that at least 1 mm of rainfall is recorded,
- S is the event that at least 1 hour of sunshine is recorded.

You are given that $P(R) = 0.28$, $P(S) = 0.87$ and $P(R \cup S) = 0.94$.

i. Find $P(R \cap S)$.

[2]

ii. Draw a Venn diagram showing the events R and S, and fill in the probability corresponding to each of the four regions of your diagram.

[3]

iii. Find $P(R \mid S)$ and state what this probability represents in this context.

[3]

6. Two events A and B are such that $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cup B) = 0.85$. Find $P(A \mid B)$. [4]

7. A particular condition affects 0.8% of the population. 90.1% of the population as a whole carry a certain gene. 9.85% of the population neither carry the gene nor are affected by the condition. Paul discovers that he carries the gene. He believes that it is very likely that he will be affected by the condition. Determine whether or not he is correct. [5]

8. In this question you must show detailed reasoning.

The probability that Judith has meat for her evening meal is 0.2 and the probability she has fish is 0.35; otherwise she has a vegetarian dish. If she has meat, the probability that she has indigestion is 0.8, if she has fish the probability that she has indigestion is 0.5 and if she has a vegetarian dish the probability that she has indigestion is 0.1.

Last night Judith had indigestion.

Calculate the probability that she had meat for her evening meal. [6]

9. In this question you must show detailed reasoning

A bag contains blue discs and red discs. There are 15 blue discs and an unknown number of red discs. There are more red discs than there are blue discs. A disc is taken at random from the bag and not replaced. A second disc is then taken at random from the bag.

Calculate the probability that 2 blue discs are taken, given that two discs of the same colour are taken. [8]

10. A is the event "Tom forgets to bring his calculator to his mathematics class".
B is the event "Tom forgets to bring his textbook to his mathematics class".

You are given that $P(A) = 0.5$ and $P(B) = 0.6$.

The probability that Tom forgets to bring both his textbook and his calculator to his mathematics class is 0.2.

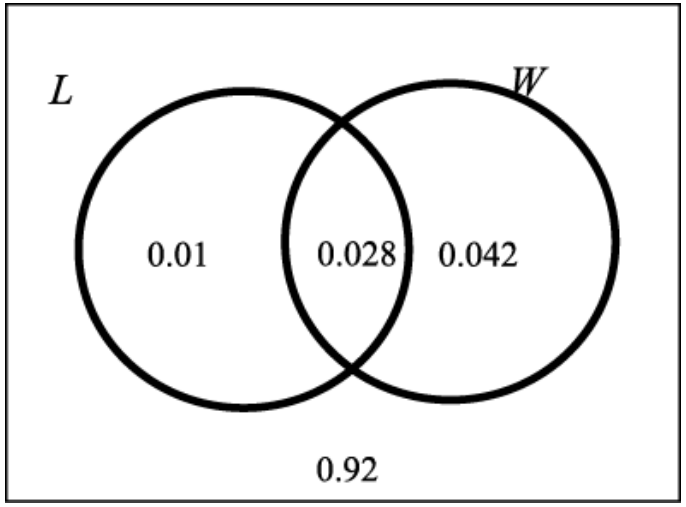
(a) Calculate $P(A|B)$. [1]

(b) Calculate $P(A|B')$. [3]

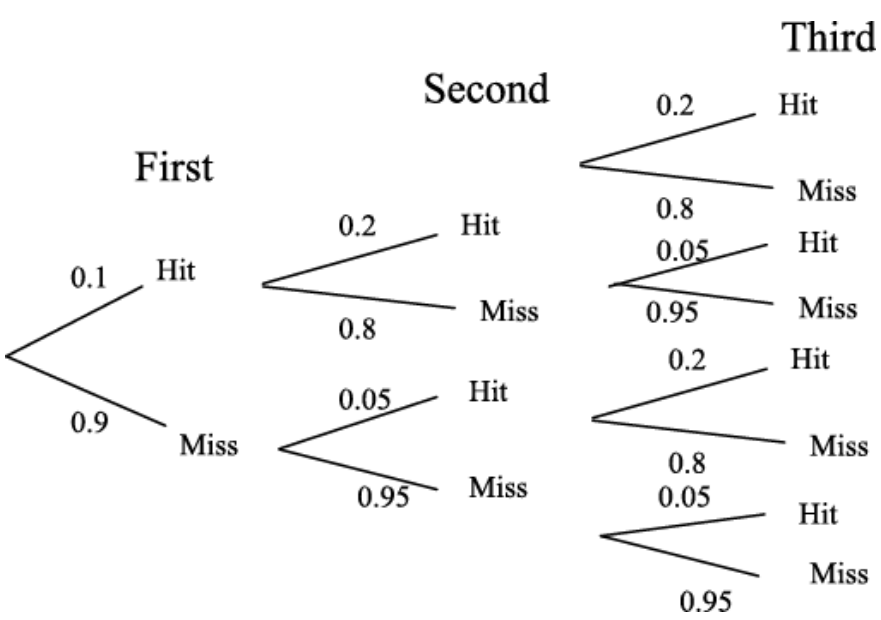
(c) State, with a reason, whether or not A and B are independent events. [1]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	$P(L \cap W) = P(L \mid W) \times P(W) = 0.4 \times 0.07 = 0.028$	M1	For $P(L \mid W) \times P(W)$ cao **Examiner's Comments** This question was well answered, with about 80% of candidates scoring both marks.	
		i		A1		
		ii		B1	For two labelled intersecting circles	
		ii		B1	For at least 2 correct probabilities. For remaining probabilities **Examiner's Comments**	FT their 0.028 provided < 0.038
		ii		B1	Almost all candidates gained the first mark for two labelled intersecting circles. Many candidates put their answer from part (i) into	

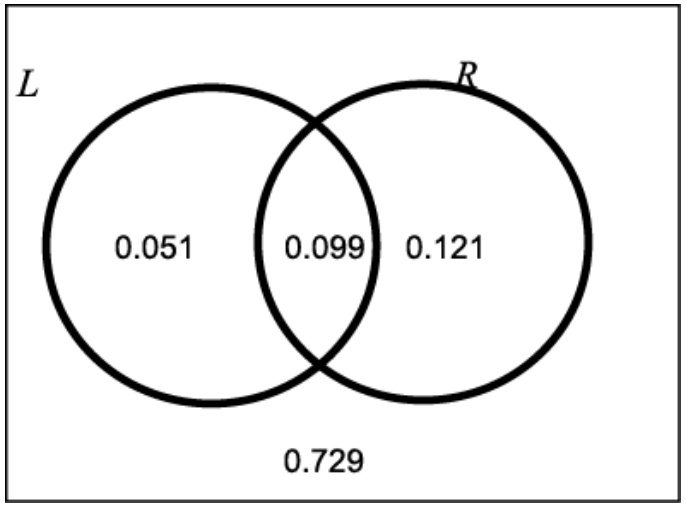
					the intersection but then did not subtract their value from P(W') so put 0.07 instead of (0.07 - their answer to (i)) in the other part of the circle labelled W. However, a reasonable number of candidates gained full credit, either having the correct 0.01 and 0.92 in the other parts or by following through correctly.	
		iii	$P(L \cap W') = 0.028$, $P(L) \times P(W') = 0.038 \times 0.07 = 0.00266$	M1	For correct use of $P(L) \times P(W')$ If P(L) wrong, max M1A0E0. No marks if P(W') wrong	Or EG $P(L \mid W') = 0.4$, $P(L) = 0.038$ Not equal so not independent M1 is for comparing with some attempt at numbers $P(L \mid W')$ with P(L), A1 for 0.038 If P(L) wrong, max M1A0E0
		iii		A1	For 0.00266	
		iii	Not equal so not independent	E1* dep on	Allow 'they are dependent' Do not award E1 if $P(L \cap W')$ wrong	
		iii		M1	**Examiner's Comments** The vast majority of candidates tried to show non-independence by comparing P(L) x P(W') with P(L intersect W'). However most of these did not have the correct value of P(L) and many had P(W') wrong, despite its value being given in the question. A small number of candidates compared $P(L \mid W')$ with P(L) and these were more often successful.	
			Total	**8**		
2		i		G1	For first set of branches	All probabilities correct

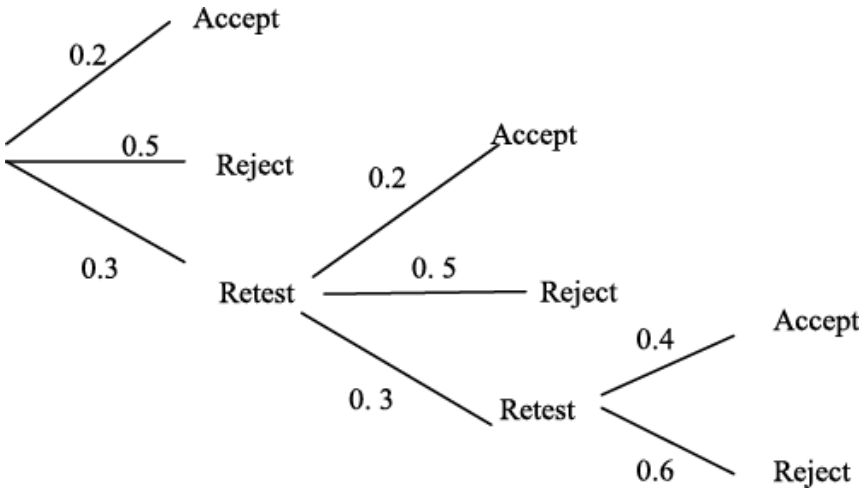
	i		G1	For second set of branches (indep)	All probabilities correct
	i		G1	For third set of branches (indep)	All probabilities correct
	i		G1	For labels Examiner's Comments The majority of tree diagrams were well constructed with correct labelling. Weaker candidates sometimes became confused and made errors in the 2nd and/or 3rd branch.	All correct labels for 'Hit' and 'Miss', 'H' and 'M' etc. Condone omission of First, Second, Third. Do not allow misreads here as all FT
	ii	P(Hits with at least one) = 1 – P(misses with all)	M1*	For 0.9 × 0.95 × 0.95	FT their tree for both M marks, provided three terms
	ii	= 1 – (0.9 × 0.95 × 0.95) = 1 – 0.81225 = 0.18775	M1*dep	For 1 – ans	
	ii		A1	CAO	0.188 or better. Condone 0.1877 Allow 751/4000
	ii	ALTERNATIVE METHOD only if there is an attempt to add 7 probabilities			
	ii	At least three correct triple products	M1		
	ii	Attempt to add 7 triple products	M1		(not necessarily correct triple products)
	ii		A1	CAO	
	ii	FURTHER ALTERNATIVE METHOD			
	ii	0.1 + 0.9 × 0.05	M1		
	ii	Above probability + 0.9 × 0.95 × 0.05	M1		

					CAO	
					Examiner's Comments	
		ii		A1	Many candidates employed the 1 – P(misses with all) method, usually successfully, but a significant number used the protracted method of listing all 7 triplets associated with at least one hit. Usually errors were made using such an approach.	
		ii	P(Hits with exactly one)	M1	For two correct products	FT their tree for all three M marks,
		ii	= (0.1 × 0.8 × 0.95) + (0.9 × 0.05 × 0.8) + (0.9 × 0.95 × 0.05)	M1	For all three correct products	provided three terms
		ii	$= 0.076 + 0.036 + 0.04275 = \frac{19}{250} + \frac{9}{250} + \frac{171}{4000}$	M1	For sum of all three correct products	
					CAO	
					Examiner's Comments	
		ii	$= \frac{619}{4000} = 0.15475$	A1	Most candidates found the correct three products and calculated them correctly. A small number failed to find all three. For those who got the tree diagram wrong, follow through marks were available.	Allow 0.155 or better
		iii	P(Hits with exactly one given hits with at least one) $= \frac{\text{P(Hits with exactly one and hits with at least one)}}{\text{P(Hits with at least one)}}$			If answer to (*B*) > than answer to (*A*) then max M1M0A0
		iii		M1	For numerator FT	Both must be part of a fraction
		iii	$= \frac{0.15475}{0.18775}$	M1	For denominator FT	

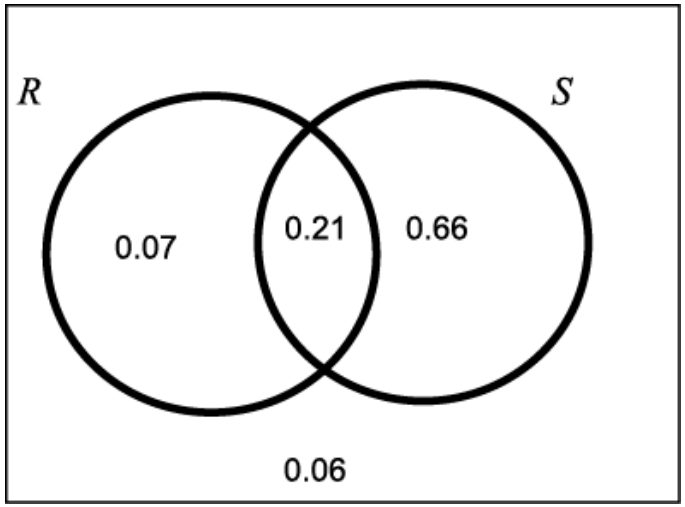
					CAO Examiner's Comments	
		iii	=0.8242	A1	Many of those who reached this part were successful. However, there was considerable confusion in finding the conditional probability, often with a correct denominator but a wrong numerator of P(at least one)×P(exactly one). Some candidates inverted the fraction.	Allow 0.824 or better or 619/751
		iv	P(Hits three times overall) = (0.1 × 0.2 × 0.2) + (0.9 × 0.95 × 0.95 × 0.05 × 0.2 × 0.2)	M1	For 0.1 × 0.2 × 0.2 or 0.004 or 1/250	FT their tree for all three M marks
		iv		M1	For 0.9 × 0.95 × 0.95 × 0.05 × 0.2 × 0.2	provided three terms in first product and six in second product. Last three probs must be 0.05 × 0.2 × 0.2 unless they extend their tree
		iv	= 0.004 + 0.0016245	M1* Dep on both prev M1's	For sum of both	With no extras Allow 0.00562 or 0.00563 or 0.0056
		iv	= 0.0056245	A1	CAO Examiner's Comments Approximately one third of candidates were successful in this part. However many were confused. Many candidates successfully found the first product but then failed to find the second, or found additional products. Those who attempted the second product often made errors. The last three probabilities were often 0.1×0.2×0.2 rather than 0.05×0.2×0.2.	NOTE RE OVER-SPECIFICATION OF ANSWERS If answers are grossly over-specified, deduct the final answer mark in every case. Probabilities should also be rounded to a sensible degree of accuracy. In general final non probability answers should not be given to more than 4 significant figures. Allow probabilities given to 5 sig fig. PLEASE HIGHLIGHT ANY OVER-SPECIFICATION

						Please note that there are no G or E marks in scoris, so use B instead
			Total	**18**		
3		i	Because P(L/R) ≠ P(L)	E1	If two or more methods given and only one correct, do not award the mark Allow 0.45 ≠ 0.15 **Examiner's Comments** The majority of candidates who scored this mark showed that P(L∩R) = 0.099 ≠ P(L) × P(R) = 0.033. Very few candidates gave the simplest explanation which is that P(L|R) ≠ P(L). For the former, candidates had to quote the correct probabilities, but for the latter the symbolic representation was adequate, as the probabilities were given in the question.	Either P($L \cap R$)(= 0.099) ≠ P(L) × P(R), provided 0.099 in (ii) or 0.099 ≠ 0.15 × 0.22 (= 0.033)
		i				Look out for complement methods, etc
		ii	P($L \cap R$) = P($L \mid R$) × P(R) = 0.45 × 0.22	M1	For product CAO **Examiner's Comments**	Allow if done correctly in part(i)
		ii	= 0.099	A1	There were three common answers here. The majority correctly obtained 0.099, but some candidates multiplied the wrong probabilities together to obtain 0.033 or 0.0675. Brief working was generally given both for the correct and the incorrect answers	Allow 99/1000

		iii		G1	For two labelled intersecting circles, provided no incorrect labelling.	Condone labels such as P(*L*) etc Allow other shapes in place of circles No need for 'box'
		iii		G1	For at least 2 correct probabilities. FT their P(*L* ∩ *R*) from part (ii) provided ≤ 0.15 For remaining probabilities. FT their P(*L* ∩ *R*) providing probabilities between 0 and 1.	FT from 0.033 in (ii) gives 0.117, 0.033, 0.187, 0.663 In general 0.15 – x, x, 0.22 – x, 0.63 + x May also see 0.0825, 0.0675, 0.1525, 0.6975
		iii		G1	**Examiner's Comments** Most candidates gained full credit here, often from a follow through of a wrong answer to part (ii). Some candidates failed to subtract P(L∩R) away from P(L) and P(R) and but were still able to score one mark for the two labelled circles.	
			Total	**6**		
4		i				Allow labels such as A, R, F(Fail) etc

		i			Do a vertical scan and give:	All probabilities correct
		i		G1	First column	All probabilities correct
		i		G1	Second column Final column Do not award if first two branches missing Branches two and three should come out of 'retest'	All probabilities correct
		i		G1	**Examiner's Comments** The vast majority of candidates were able to correctly construct the tree diagram although it did appear that quite a few needed two attempts (it looked as though there made been some rubbing out under the final version). Only a very small number of candidates omitted any of the required labels or mixed up some of the probabilities, but	If any labels missing or incorrect allow max 2/3 Do not allow misreads here as all FT (eg 0.3 and 0.5 reversed)

					these candidates were able to gain follow through marks in subsequent parts of the question. A few candidates omitted the middle set of branches, or added extra sets following 'Accept' or 'Reject'.	
		ii	P(Accepted) = 0.2 + (0.3 × 0.2) + (0.3 × 0.3 × 0.4)	M1	For second or third product	FT their tree provided correct numbers of terms and correct structure of 3, 3, 2 branches.
		iii	= 0.2 + 0.06 + 0.036 = 0.296	A1	CAO **Examiner's Comments** This was generally very well answered.	Allow 37/125 oe
		iv	P(At least one retest given accepted)			
		iv	$= \frac{\text{P(At least one retest and accepted)}}{\text{P(Accepted)}}$	M1	For numerator	FT their tree provided correct numbers of terms and correct structure of 3, 3, 2 branches. for both M1's
		iv	$= \frac{(0.3 \times 0.2) + (0.3 \times 0.3 \times 0.4)}{0.296} = \frac{0.096}{0.296}$	M1	For denominator FT their 0.296 and 0.096 Allow 0.32 with working **Examiner's Comments**	Both must be part of a fraction Allow 12/125 oe
		iv	= 0.324	A1	Candidates found this part much more difficult and many gave an answer of 0.096, which is simply the probability that a candidate for the job is retested at least once and accepted, so not a conditional probability at all. This scored zero unless it was as the numerator of a fraction. Other candidates did	Allow 12/37 oe

					have a fraction with the correct denominator but their numerator was incorrect.	
			Total	**8**		
5		i	$P(R \cap S) = P(R) + P(S) - P(R \cup S)$ $= 0.28 + 0.87 - 0.94$	M1	For correct use of formula	Or $0.28 - x + 0.87 - x + x = 0.94$
		i	$= 0.21$	A1		
		i			**Examiner's Comments** This part was very well answered but a considerable number of candidates assumed that the probabilities were independent and calculated $P(R) \times P(S)$. Some were more confused about the correct formula to use and calculated $P(R) \times P(R \cup S)$.	
		ii		G1	For two labelled intersecting circles	Allow labels such as $P(R)$ and $P(S)$ Allow other sensible shapes in
		ii		G1	For at least 2 correct probabilities. FT their $P(R \cap S)$	place of circles Allow their $P(R \cap S)$ rounded to 2dp For both G1 marks FT their 0.21

					For remaining probabilities. FT their P($R \cap S$) **Examiner's Comments**	**provided < 0.28**
		ii		G1	The idea of the Venn diagram was well understood, and most candidates produced a fully correct solution (often following through from an error in part (i)). Very few noticed the contradiction produced by their wrong answer, which gave the outer zone as 0.0936 instead of 0.06 from the question.	For FT if P($R \cup S$) = x then others are $0.28 - x$, $0.87 - x$, $x - 0.15$ 0.2436 leads to 0.0364, 0.6264, 0.0936
		iii	$P(R \mid S) = \dfrac{P(R \cap S)}{P(S)} = \dfrac{0.21}{0.87} = \dfrac{21}{87} = 0.241$	M1	for fraction	Allow $\dfrac{7}{29}$ or $\dfrac{21}{87}$ as final answer
		iii	Exact answer 0.241379…	A1	CAO FT their part (i) (for M1 only) but M0 if their answer to part (i) is P(R) × P(S)	Allow 0.24 with working Condone 'if' or 'when' for 'given that' but not the words 'and' or 'because' or 'due to' for E1.
		iii	This is the probability that (on a randomly selected day) there is at least 1 mm of rain, given that there is at least 1 hour of sun.	E1	Need more than just probability of rain given sun Must include 'probability' or 'chance' oe Do not allow just P(at least 1 mm of rain, given that there is at least 1 hour of sun) **Examiner's Comments** Among those who had not made the independence error in part (i), the correct answer was quite common. The explanation of what the probability means was usually correct but sometimes lacked sufficient detail. There were a few candidates who 'reversed'	E1 (independent of M1): the order / structure must be correct i.e. no reverse statement Allow 'The probability that on a randomly selected day when there is at least 1 hour of sun there is at least 1 mm of rain.' oe

				the statement and gave an explanation of P(S|R).		
			Total	**8**		
6			$P(A \cap B) = P(A) + P(B) - P(A \cup B)$ $= 0.6 + 0.5 - 0.85$ $= 0.25$ $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$ $= \frac{0.25}{0.5}$ $= 0.5$	M1(AO3.1a) A1(AO1.1) M1(AO1.1) A1(AO1.1) [4]		
			Total	**4**		
7			p(G or C) [= 1 – 0.0985] = 0.9015 P(G and C) = 0.901 + 0.008 – 0.9015[= 0.0075] $P(C/G) = \frac{their\ 0.0075}{0.901}$ = 0.00832 This is very unlikely so he is incorrect.	B1(AO 3.1a) M1(AO 1.1) M1(AO 2.1) A1(AO 1.1) E1(AO 3.2a) [5]	P(G or C) = P(G) + P(C) - P(G and C) used	

			Total	**5**			
8			[0.8 × 0.2 =] 0.16 seen 0.35 × 0.5 and 0.45 × 0.1 seen Finds sum of their attempt at these probabilities 0.38 *their* 0.16 ÷ *their* 0.38 $\frac{8}{19}$	B1(AO1.1b) B1(AO3.1b) M1(AO3.1a) A1(AO2.1) M1(AO1.1b) A1(AO3.1a) [6]	or tree diagram with all necessary outcomes and associated probabilities may be implied by 0.38 0.42105263… rounded to 3 sf or more	allow omission of 0.2, 0.5 and 0.9 together with labelling	
			Total	**6**			
9			$\frac{15}{n}\times\frac{14}{n-1}+\frac{n-15}{n}\times\frac{n-1-15}{n-1}=2\times\frac{15}{n}\times\frac{n-15}{n-1}$ Multiply through to obtain quadratic in *n*	M1 (AO3.1b) M1 (AO2.1) A1 (AO1.1) M1 (AO2.1) A1 (AO1.1) B1 (AO3.2b)	P(BB) + P(RR) = P(one of each) Two of three terms correct	*n* is the total number of discs	

$n2 - 61n + 900 = 0$

$n = 36$ (not 25) since more red discs

$$\frac{\frac{15}{36}\times\frac{14}{35}}{\frac{15}{36}\times\frac{14}{35}+\frac{21}{36}\times\frac{20}{35}}$$

$1/3$

OR

$$\frac{15}{15+r}\times\frac{14}{14+r}+\frac{r}{15+r}\times\frac{r-1}{14+r}=2\times\frac{15}{15+r}\times\frac{r}{14+r}$$

Multiply through to obtain quadratic in r

$r^2 - 31r + 210 = 0$

$r = 21$ (not 10) since $r > 15$

$$\frac{\frac{15}{36}\times\frac{14}{35}}{\frac{15}{36}\times\frac{14}{35}+\frac{21}{36}\times\frac{20}{35}}$$

$1/3$

M1 (AO2.1)

A1 (AO1.1)
[8]

M1
M1
A1

M1

A1

B1

M1

A1
[8]

All correct	
so there are 21 red discs	
FT their 44	
Accept decimal equivalent to 2 or more dp	r is the number of red discs
P(BB) + P(RR) = P(one of each) Two of three terms correct All correct	

					FT their 21 (> 15) Accept decimal equivalent to 2 or more dp	
			Total	**8**		
10		a	$\frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.6} = \frac{1}{3}$	B1(AO 1.1) [1]	Accept use of Venn diagram	
		b	$P(A \cup B) = 0.9$ $P(A \cap B') = 0.9 - 0.6 = 0.3$ $\frac{0.3}{0.4} = 0.75$	B1(AO 2.1) B1(AO 1.1) B1(AO 1.1) [3]	Accept use of Venn diagram	
		c	$\frac{1}{3} \neq \frac{3}{4}$ or $\frac{1}{3} \neq \frac{1}{2}$ or $\frac{3}{4} \neq \frac{1}{2}$ so not independent	B1(AO 2.2a) [1]		
			Total	**5**		